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Intuitionistic (α, β)- Fuzzy H_v-Subgroups

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Abstract

Atanassov introduced the notion of intuitionistic fuzzy sets as a generalization of the notion of fuzzy sets. In this paper we introduce the concept of an intuitionistic (α, β)-fuzzy H_v-subgroups of an H_v-groups by using the notion of "belongingness $(\in)$" and "quasi-coincidence (q)" of fuzzy points with fuzzy sets, where $\alpha \in \{\in, q\}$, $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and, then we investigate the basic properties of these notions.

Mathematics Subject Classification: 20N20

Keywords: H_v-group, H_v-subgroup, fuzzy H_v-group, fuzzy H_v-subgroup, intuitionistic (α, β) - fuzzy H_v-subgroup.

Introduction

The concept of hyperstructure was introduced in 1934 by Marty [1]. Hyperstructures have many applications to several branches of pure and applied sciences. Vougiouklis [2] introduced the notion of H_v-structures, and Davvaz [3] surveyed the theory of H_v-structures. After the introduction of fuzzy sets by Zadeh [4], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [5] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [6, 7].

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [14], played a vital role to generate some different types of fuzzy subgroups. Bhakat and Das [8, 9] gave the concepts of (α, β)-fuzzy subgroups by using the notion of "belongingness ($\in$)" and "quasi-coincidence (q)" between a fuzzy point and a fuzzy subgroup, where α, β are any two of $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$, and introduced the concept of an $(\in, \in \vee q)$-fuzzy subgroup. In [10] Yuan, Li et al. redefined (α, β)-intuitionistic fuzzy subgroups. M. Asghari-Larimi [15] gave intuitionistic (α, β)-fuzzy H_v-submodules. This paper continues this line of research for fuzzy H_v-subgroups of H_v-groups.

The paper is organized as follows: in section 2 some fundamental definitions on H_v-structures and fuzzy sets are explored, in section 3 we define intuitionistic (α, β)-fuzzy H_v-subgroups and establish some useful theorems.

Basic Definitions

We first give some basic definitions for proving the further results.

Definition 2.1 [11] Let X be a non-empty set. A mapping $\mu : X \to [0, 1]$ is called a fuzzy set in X.

Definition 2.2 [11] An intuitionistic fuzzy set A in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\lambda_A : X \to [0, 1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \lambda_A(x) \le 1$ for all $x \in X$. We shall use the symbol $A = \{\mu_A, \lambda_A\}$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$.

Definition 2.3 [12] Let H be a non–empty set and $* : H \times H \to \wp^*(H)$ be a hyperoperation, where $\wp^*(H)$ is the set of all the non-empty subsets of H. Where $A * B = \bigcup_{a \in A, b \in B} a * b, \ \forall A, B \subseteq H$.

The $*$ is called weak commutative if $x * y \cap y * x \ne \phi, \quad \forall x, y \in H$.
The $*$ is called weak associative if $(x * y) * z \cap x * (y * z) \ne \phi, \quad \forall x, y, z \in H$.

$(H, *)$ is called an H_v–group if
(i) $*$ is weak associative.
(ii) $a * H = H * a = H, \quad \forall a \in H$ (Reproduction axiom).

Definition 2.4 [13] Let H be a hypergroup (or H_v-group) and let μ be a fuzzy subset of H. Then μ is said to be a fuzzy subhypergroup (or fuzzy H_v-subgroup) of H if the following axioms hold:

$(i) \min\{\mu(x), \mu(y)\} \le \inf_{\alpha \in x*y} \{\mu(\alpha)\}, \quad \forall x, y \in H.$

(ii) For all $x, a \in H$ there exists $y \in H$ such that $x \in a * y$ and $\min\{\mu(a), \mu(x)\} \le \{\mu(y)\}$.

Definition 2.5 [5] Let $A = \{\mu_A, \lambda_A\}$ and $B = \{\mu_B, \lambda_B\}$ be intuitionistic fuzzy sets in X. Then $(1)\ A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x) \text{ and } \lambda_A(x) \le \lambda_B(x)\ \forall x \in X$,

$(2)\ A^c = \{(x, \lambda_A(x), \mu_A(x)) : x \in X\}$,

$(3)\ A \cap B = \left\{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}) : x \in X\right\}$,

$(4)\ A \cup B = \left\{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}) : x \in X\right\}$

Definition 2.6 [8] Let μ be a fuzzy subset of R. If there exist a $t \in (0,1]$ and an $x \in R$ such that

$$\mu(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{if } y \ne x \end{cases}$$

Then μ is called a fuzzy point with support x and value t and is denoted by x_t.

Definition 2.7 [8] Let μ be a fuzzy subset of R and x_t be a fuzzy point.

(1) If $\mu(x) \ge t$, then we say x_t belongs to μ, and write $x_t \in \mu$.

(2) If $\mu(x) + t > 1$, then we say x_t is quasi-coincident with μ, and write $x_t q \mu$.

(3) $x_t \in \vee q \mu \Leftrightarrow x_t \in \mu$ or $x_t q \mu$.

(4) $x_t \in \wedge q \mu \Leftrightarrow x_t \in \mu$ and $x_t q \mu$.

In what follows, unless otherwise specified, α and β will denote any one of $\in, q, \in \vee q$ or $\in \wedge q$ with $\alpha \ne \in \wedge q$, which was introduced by Bhakat and Das [9].

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Intuitionistic (α, β) - fuzzy H$_v$-subgroups

In this section we give the definition of intuitionistic (α, β)-fuzzy H$_v$-subgroup and prove some related results.

Definition 3.1 Let H be a hypergroup (or H$_v$-group). An intuitionistic fuzzy set $A=\{\mu_A, \lambda_A\}$ of H is called intuitionistic fuzzy subhypergroup (or intuitionistic fuzzy H$_v$-subgroup) of H if the following axioms hold:

(i) $\min\{\mu(x), \mu(y)\} \le \inf_{\alpha\in x*y}\{\mu(\alpha)\}, \quad \forall x, y\in H.$

(ii) For all $x, a\in H$ there exists $y\in H$ such that $x\in a*y$ and $\min\{\mu(a), \mu(x)\} \le \{\mu(y)\}$.

(iii) $\sup_{\alpha\in x*y}\{\lambda_A(\alpha)\} \le \max\{\lambda_A(x), \lambda_A(y)\}, \quad \forall x, y\in H.$

(iv) For all $x, a\in H$ there exists $y\in H$ such that $x\in a*y$ and $\{\lambda_A(y)\} \le \max\{\lambda_A(a), \lambda_A(x)\}$.

Definition 3.2 An intuitionistic fuzzy set $A=\{\mu_A, \lambda_A\}$ in G is called an intuitionistic (α, β)-fuzzy H_v-subgroup of G if for all $t, r\in (0,1]$,

(1) $\forall x, y\in G, \quad x_t, y_r \alpha\, \mu_A \Rightarrow z_{t\wedge r}\beta\, \mu_A$ for all $z\in x\cdot y$,

(2) $\forall x, a\in G, \quad x_t, a_r \alpha\, \mu_A \Rightarrow y_{t\wedge r}\beta\, \mu_A$ for some $y\in G$ with $x\in a\cdot y$,

(3) $\forall x, y\in G, \quad x_t, y_r \overline{\alpha}\, \lambda_A \Rightarrow z_{t\wedge r}\overline{\beta}\, \lambda_A$ for all $z\in x\cdot y$,

(4) $\forall x, a\in G, \quad x_t, a_r \overline{\alpha}\, \lambda_A \Rightarrow y_{t\wedge r}\overline{\beta}\, \lambda_A$ for some $y\in G$ with $x\in a\cdot y$,

Lemma 3.3 Let $A=\{\mu_A, \lambda_A\}$ be an intuitionistic fuzzy set in G. Then for all $x\in G$ and $r\in (0,1]$, we have

(1) $x_t q\mu_A \Leftrightarrow x_t \overline{\in} \mu_A^c.$

(2) $x_t \in\vee q\mu_A \Leftrightarrow x_t \overline{\in\wedge q}\, \mu_A^c.$

Proof. (1) Let $x\in G$ and $r\in (0,1]$. Then, we have

$$\begin{aligned} x_t q\mu_A &\Leftrightarrow \mu_A(x)+t>1 \\ &\Leftrightarrow 1-\mu_A(x)<t \\ &\Leftrightarrow \mu_A^c(x)<t \\ &\Leftrightarrow x_t \overline{\in} \mu_A^c. \end{aligned}$$

(2) Let $x\in G$ and $r\in (0,1]$. Then, we have

$x_t \in\vee q\mu_A \Leftrightarrow x_t\in\mu_A$ or $x_t q\mu_A$ $\Leftrightarrow \mu_A(x)\ge t$ or $\mu_A(x)+t>1$ $\Leftrightarrow 1-\mu_A^c(x)\ge t$ or $1-\mu_A^c(x)+t>1$ $\Leftrightarrow x_t\overline{q}\mu_A^c$ or $x_t \overline{\in} \mu_A^c$

$\Leftrightarrow x_t \overline{\in\wedge q}\mu_A^c.$

Theorem 3.4 If $A=\{\mu_A, \lambda_A\}$ is an intuitionistic $(\in,\in)$-fuzzy H_v-subgroup of G, then $A=\{\mu_A, \lambda_A\}$ is an intuitionistic fuzzy H_v-subgroup of G.

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Proof (1) Let $x, y \in G$ and $\mu_A(x) \wedge \mu_A(y) = t$. Then $x_t, y_t \in \mu_A$. By condition (1) of definition 3.2, we have $z_t \in \mu_A, \quad \forall z \in x \cdot y$, and so $\mu_A(z) \geq t, \quad \forall z \in x \cdot y$.

Consequently $\mu_A(x) \wedge \mu_A(y) = t \leq \bigwedge_{z \in x \cdot y} \mu_A(z)$ for all $x, y \in G$.

(2) Now let $x, a \in G$ and $\mu_A(x) \wedge \mu_A(a) = t$. Then $x_t, a_t \in \mu_A$. It follows from condition (2) of definition 3.2 that $y_t \in \mu_A$, for some $y \in G$ with $x \in a \cdot y$.

Thus $y_t \in \mu_A$, for some $y \in G$ with $x \in a \cdot y$.

So, for all $x, a \in G$, there exist $y \in G$ such that $x \in a \cdot y$ and $\mu_A(x) \wedge \mu_A(a) = t \leq \mu_A(y)$.

(3) Let $x, y \in G$ and $\lambda_A(x) \vee \lambda_A(y) = s$. If $s = 1$, then $\lambda_A(z) \leq 1 = s$ for all $z \in x \cdot y$. It is easy to see that $\bigvee_{z \in x \cdot y} \lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$ for all $x, y \in G$

If $s < 1$ there exists a $t \in (0,1]$ such that $\lambda_A(x) \vee \lambda_A(y) = s < t$

Then $x_t, y_t \overline{\in} \lambda_A$. By condition (3) of definition 3.2, we have $z_t \overline{\in} \lambda_A, \quad \forall z \in x \cdot y$ and so $\lambda_A(z) < t$.

Consequently $\bigvee_{z \in x \cdot y} \lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$ for all $x, y \in G$.

(4) Now let $x, a \in G$ and $\lambda_A(x) \vee \lambda_A(a) = s$. If $s < 1$, there exists a $t \in (0,1]$ such that $\lambda_A(x) \vee \lambda_A(a) = s < t$.

Then $x_t, a_t \overline{\in} \lambda_A$. By condition (4) of definition 3.2, we have $y_t \overline{\in} \lambda_A$ for some $y \in G$ with $x \in a \cdot y$

Hence $\lambda_A(y) < t$ and $\lambda_A(z) < t$.

Thus $\lambda_A(y) \vee \lambda_A(z) < t$. This implies that for all $x, a \in G$, there exist $y \in G$ such that $x \in a \cdot y$ and $\lambda_A(y) \leq \lambda_A(x) \vee \lambda_A(a)$. If $s = 1$ the proof is obvious.

Theorem 3.5 If $A = \{\mu_A, \lambda_A\}$ is an intuitionistic $(\in, \in \vee q)$ and $(\in, \in \wedge q)$-fuzzy H_v-subgroup of G, then $A = \{\mu_A, \lambda_A\}$ is an intuitionistic fuzzy H_v-subgroup of G.

Proof The proof is similar to the proof of Theorem 3.4.

Theorem 3.6 If □$A = \{\mu_A, \mu^c{}_A\}$ is an intuitionistic (α, β)-fuzzy H_v-subgroup of G if and only if □$A = \{\mu_A, \mu^c{}_A\}$ is an intuitionistic (α', β')-fuzzy H_v-subgroup of G, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.

Proof We only prove the case of $(\alpha, \beta) = (\in, \in \vee q)$. The others are analogous. Let □$A = \{\mu_A, \mu^c{}_A\}$ be an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-subgroup of G.

Condition (1). Let $x, y \in G$ and $t, r \in (0,1]$ be such that $x_t, y_r q \mu_A$. It follows from Lemma 3.3 that $x_t, y_r \overline{\in} \mu_A^c$. Since μ_A^c is an anti $(\in, \in \vee q)$-fuzzy H_v-subgroup of G. Thus by condition (3) of definition 3.2, we have

$$z_{t \wedge r} \overline{\in \vee q} \mu_A^c \text{ for all } z \in x \cdot y.$$

By Lemma 3.3, this is equivalence with

$z_{t \wedge r} \in \wedge q \mu_A$ for all $z \in x \cdot y$.

Thus condition of (1) of definition 3.2 is valid.

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Condition (2). Suppose that $x, a \in G$ and $t, r \in (0,1]$ be such that $x_t, a_r q \mu_A$. By Lemma 3.3, we have $x_t, a_r q \mu_A$ iff $x_t, a_r \overline{\in} \mu_A^c$. By hypotheses, μ_A^c is an anti $(\in, \in \vee q)$-fuzzy H_v-subgroup of G. Thus by condition (4) of definition 3.2, we have

$y_{t\wedge r} \overline{\in \vee q} \mu_A^c$ for some $y \in G$ with $x \in a \cdot y$.

It follows from Lemma 3.2 that

$y_{t\wedge r} \in \wedge q \mu_A$ for some $y \in G$ with $x \in a \cdot y$.

Thus condition of (2) of definition 3.2 is valid.

Condition (3). Let $x, y \in G$ and $t, r \in (0,1]$ be such that $x_t, y_r \overline{q} \mu_A^c$. It follows from Lemma 3.3 that $x_t, y_r \overline{q} \mu_A^c$ iff $x_t, y_r \in \mu_A$. Since $\square A = \{\mu_A, \mu^c{}_A\}$ is an intuitionistic $(\in, \in \vee q)$-fuzzy H_v-subgroup of G .Thus by condition (1) of definition 3.2, we have

$$z_{t\wedge r} \in \vee q \mu \text{ for all } z \in x \cdot y.$$

By Lemma 3.2, this is equivalence with

$$z_{t\wedge r} \overline{\in \wedge q} \mu_A^c \text{ for all } z \in x \cdot y.$$

Thus condition of (3) of definition 3.2 is valid.

Condition (4). Suppose that $x, a \in G$ and $t, r \in (0,1]$ be such that $x_t, a_r \overline{q} \mu_A^c$. This is equivalence with $x_t, a_r \in \mu_A$. By hypotheses, μ_A is an $(\in, \in \vee q)$-fuzzy H_v-subgroup of G. Thus by condition (2) of definition 3.2, we have

$y_{t\wedge r} \in \vee q \mu_A$ for some $y \in G$ with $x \in a \cdot y$.

It follows from Lemma 3.3 that

$y_{t\wedge r} \overline{\in \wedge q} \mu_A^c$ for some $y \in G$ with $x \in a \cdot y$.

Thus condition of (4) of definition 3.2 is valid.

Theorem 3.7 If $\lozenge A = \{\lambda_A^c, \lambda_A\}$ is an intuitionistic (α, β)-fuzzy H_v-subgroup of G if and only if $\lozenge A = \{\lambda_A^c, \lambda_A\}$ is an intuitionistic (α', β')-fuzzy H_v-subgroup of G, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.

Proof The proof is similar to the proof of Theorem 3.6.

Theorem 3.8 If $A = \{\mu_A, \lambda_A\}$ is an intuitionistic (α, β)-fuzzy H_v-subgroup of G if and only if μ_A is an (α, β)-fuzzy H_v-subgroup of G and λ_A^c is an (α', β')-fuzzy H_v-subgroup of G, where $\alpha \in \{\in, q\}$ and $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$.

Proof We only prove the case of $(\alpha, \beta) = (\in, \in \vee q)$. The others are analogous. It is sufficient to show that, λ_A^c is an $(q, \in \wedge q)$-fuzzy H_v-subgroup of G if and only if λ_A is an anti $(\in, \in \vee q)$-fuzzy H_v-subgroup of G .This is true, because $x_t q \lambda_A \Leftrightarrow x_t \overline{\in} \lambda_A^c$ and $x_t \in \wedge q \lambda_A \Leftrightarrow x_t \overline{\in \vee q} \lambda_A^c$, $\forall x \in G$ and $t \in (0,1]$.

References

[1] *F. Marty, Sur une generalization de la notion de group, in: 8th congress Math.Skandenaves, Stockholm, 1934, pp. 45-49.*

[2] *T. Vougiouklis, A new class of hyperstructures, J. Combin. Inf. System Sci., to appear.*

[3] *B. Davvaz, A brief survey of the theory of H_v-structures, in: Proceedings of the 8^{th} International Congress on AHA, Greece 2002, Spanids Press, 2003, pp. 39-70.*

[4] *L. A. Zadeh, Fuzzy sets, Inform. And Control 8 (1965) 338-353.*

[5] *K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.*

ISSN: 2277-9655
Impact Factor: 1.852

[6] K. T. Atanassov, New operations defined over the intuitionistic fuzzy sets, Fuzzy Sets and Systems 61, (1994) 137-142.
[7] K. T. Atanassov, Intuitionistic fuzzy sets: Theory and Applications, Studies in fuzziness and soft computing, 35, Heidelberg, New York, Physica-Verl., 1999.
[8] S. K. Bhakat, P. Das, On the definition of a fuzzy subgroup, Fuzzy Sets and Systems, 51 (1992) 235-241.
[9] S. K. Bhakat, P. Das, $(\in, \in \vee q)$ -fuzzy subgroups, Fuzzy Sets and Systems, 80 (1996) 359- 368.
[10]X. H. Yuan , H. X. Li and E. S. Lee, On the definition of the intuitionistic fuzzy subgroups, Computers and Mathematics with Applications, 59 (2010) 3117-3129.
[11]B. Davvaz, W. A. Dudek, Y. B. Jun, Intuitionistic fuzzy H_v–submodules, Inform. Sci. 176 (2006) 285-300.
[12]T. Vougiouklis, Hyperstructures and their representations, Hadronic Press, Florida, 1994. [13] B. Davvaz, Fuzzy H_v-groups, Fuzzy Sets and Systems 101 (1999) 191-195.
[13]B. Davvaz, Fuzzy Hv-groups, Fuzzy Sets and Systems 101 (1999) 191-195.
[14]P. M. Pu and Y. M. Liu, Fuzzy topology: Neighourhood structure of a fuzzy point and Moore-Smooth convergence, J. Math. Anal. Appl., 76 (1980) 571-599.
[15]M. Asghari-Larimi, Intuitionistic (α, β)-Fuzzy H_v-Submodules, J. Math. Comput. Sci. 2 (2012), No. 1, 1-14.
[16]